THE COMBINED EFFECT OF MAGNETIC FLUIDS WITH COUPLE STRESSES, VARIABLE VISCOSITY AND VELOCITY-SLIP ON THE LUBRICATION OF FINITE JOURNAL BEARINGS.

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Abstract: In this paper, we examine the combined effect of a non-Newtonian couple-stress lubricant, and a magnetic fluid, together with velocity-slip and piezo-viscosity, has on the lubrication characteristics of a finite journal bearing. Using the Stokes micro-continuum theorem and the Barus formula with an artificial (homogeneous) slip surface, we investigate the load-carrying ability, pressure distribution, and frictional coefficient of the bearing. Our results indicate that the piezo-viscosity parameter improves the maximum magnetic and hydrodynamic pressures of the journal bearing. The combined effect also significantly enhances the bearing characteristics.

Keywords: couple-stress lubricant, finite journal bearing, magnetic fluid, variable viscosity, velocity-slip.

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1. Introduction

The ferrofluid (magnetic fluid) is composed of three main constitutive parts; the single-domain magnetic particles, the carrier fluid (base fluid), and the surfactant. Every magnetic particle is coated with a layer of long-chain or polymer molecules. The surfactant layer keeps the magnetic particles at a distance so that the attraction between particles caused by the magnetic force avoids aggregation of the fluid. A growing body of literature has examined the field of ferrofluid lubricated bearings ([1]-[6]). Research has shown that a ferrofluid lubricant increases the pressure and the load-carrying capacity of the bearings. The bearing stability, wear, friction, noise and viscosity are enhanced. Patel and Deheri [7] studied the effect of slip velocity and the short journal bearing lubricated with a magnetic fluid. Osman et al. [8] investigated the effect of using a current-carrying-wire model in the design of hydrodynamic journal bearings lubricated with ferrofluid. J.R. Lin et al. [9], Hanumagowda et al. [10] investigated journal bearings lubricated with couple stress fluids and pressure-dependent viscosity.

The Barus formula was used to define the variable viscosity. Rao et al. [11] performed work based on a theoretical model of partially textured, slip, slider and journal bearing lubricated with couple stress fluids. The authors found that the load carrying capacity increased with the inclusion of slip and the couple-stress parameter. Friction reduction also occurred with increasing slip and couple-stress. N.C. Das, [12], Nada and Osman, [13] conducted a study to ascertain the optimum load bearing capacity for slider bearings lubricated with couple stress fluids in a magnetic field. Researchers discovered that the maximum load capacity and the pressure distribution increases with the increase of both magnetic and couple stress parameters. J.R. Lin et al. [14] investigated the ferrofluid model of Shliomis and the micro-continuum theory of Stokes, taking into account the effects of rotation of ferromagnetic particles and the effects of non-Newtonian properties. Rao and Prasad [15] studied the effect of velocity-slip and viscosity variation on journal bearings and found that the load carrying capacity of the bearing was less when slip was present also, friction decreased with viscosity. Oladeinde and Akpobi [16] investigated the load capacity of slider bearings with slip surfaces and couple-stress fluids. The finite element method was used and it was concluded that the couple-stress parameter causes an increase in the load capacity and to achieve maximum load the slip-velocity must attain some minimum value. Dass et al. 2019 [17] investigated the combined effect of variable viscosity, couple-stress lubricant with three cases of slip. It was found that the engineered slip on the bearing and no-slip on the journal yielded the greatest enhancement in load carrying ability of the journal bearing and pressure distribution. The combined effect also reduced the friction. The attitude angle was significantly decreased in the presence of these augmented parameters. In our present study, we consider the combined effect of variable viscosity, couple-stress fluid, ferrofluid, and slip-velocity on the journal bearing. We examine how this combination of parameters affects the pressure distribution, load carrying capacity, and friction of the journal bearing. This study is an attempt to configure the ideal journal bearing.

1.1 Mathematical Formulation

1.1.1 Bearing Geometry

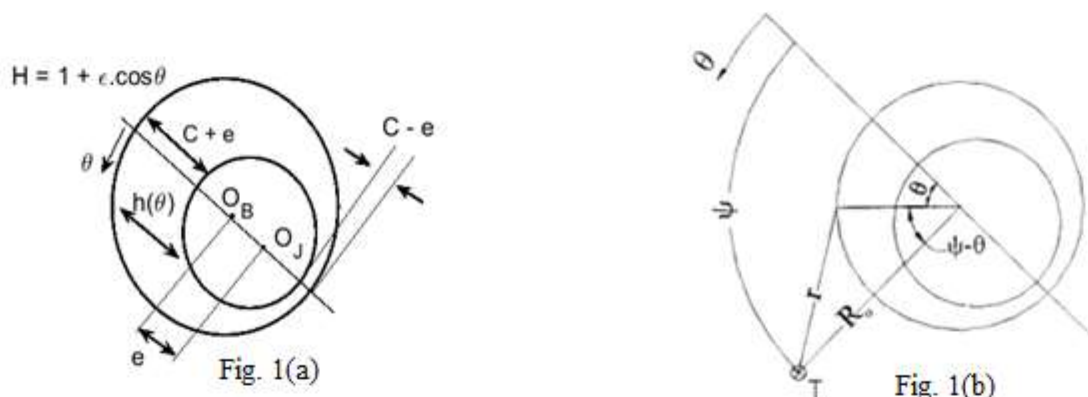

Figure 1

As shown in Fig. 1, the magnetic field is produced by a current passing through an infinitely long wire that is displaced a distance (R_0) greater than bearing radius. The wire is placed at an angle ψ from the line of the journal and bearing centers to obtain enhanced pressure distribution Tarapov[18] . Figure 1(b) represents the displaced infinitely long wire magnetic field model, which has a field gradient only in the circumferential direction of the bearing. The Barus formula for the variation of the viscosity with pressure is given as:

$$\mu = \mu_0 e^{\alpha p} \qquad (1)$$

The induced magnetic force for the ferrofluid in the presence of the magnetic field is given by Cowley and Rosensweig [19], Zelazo and Melchier [20]

$$F_m = \mu_0 X_m h_m \left(\nabla h_m\right) \qquad (2)$$

F_m will be used as an external body force in the field equations Zang [21]. From the Stoke's micro-continuum theory the general momentum equation for an incompressible fluid with couple stress is given as:

$$\rho \frac{d\vec{u}}{dt} = -\nabla P + \rho \vec{F}_m + \frac{1}{2}\rho \nabla \times \vec{C} + \mu \Delta \vec{u} - \eta \Delta^2 \vec{u}. \tag{3}$$

Here $\vec{u}$, $\vec{F}_m$ and $\vec{C}$ represent the velocity, body force per unit mass, and the body couple per unit mass respectively. P is the pressure, ρ is the density, μ is the shear viscosity, and η is a material constant responsible for the couple stress property. This couple stress parameter characterizes the effects of couple stresses on the bearing characteristics of the system. Starting from the Navier-Stokes equations, and using the magnetic force as an external body force, the equations of motion are derived for the fluid film. It is assumed that the body couples are absent for an incompressible fluid, the lubricant is non-Newtonian with constant density and slip on the journal. The field equation in the *x*-direction then becomes:

$$\eta \frac{\partial^4 u}{\partial z^4} - \frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + F_{mx}. \tag{4}$$

1.2 Boundary Conditions

The boundary conditions with slip on the journal are

$$u = U,\ \frac{\partial^2 u}{\partial z^2} = 0,\ \text{at } z = 0;\ u = -\lambda\mu\left(\frac{\partial u}{\partial z}\right) \text{ and } \left(\frac{\partial^2 u}{\partial z^2}\right) = 0 \text{ at } z = h \tag{5}$$

Using the boundary conditions in the field equation we obtain the velocity in the *x*-direction:

$$u = U - \frac{e^{-\alpha p}}{\mu_0}\frac{U}{\left(\lambda + \frac{e^{-\alpha p}}{\mu_0}h\right)}z + \frac{e^{-\alpha p}}{\mu_0}\left(\frac{dp}{dx} - F_{mx}\right)\times \left(z^2 - \frac{\frac{e^{-\alpha p}}{\mu_0}h^2 + 2\lambda h - 2\lambda l e^{-\alpha p} tanh\left(\frac{e^{-0.5\alpha p}}{2l}h\right)}{\left(\lambda + \frac{e^{-\alpha p}}{\mu_0}h\right)}z\right) + \frac{e^{-\alpha p}}{\mu_0}\left(\frac{dp}{dx} - F_{mx}\right)\left(2l^2 e^{-\alpha p}\left\{1 - \frac{cosh\left(e^{\alpha p}\frac{(2z-h)}{2l}\right)}{cosh\left(e^{0.5\alpha p}\frac{h}{2l}\right)}\right\}\right) \tag{6}$$

1.3 Modified Reynolds Equation

With the help of Eq. (6) and the continuity equation we obtain the following Modified Reynolds Equation by integrating with respect to z from $z = 0$ to $z = h$.

$$\frac{\partial}{\partial x}\left[\frac{h^3 e^{-\alpha p}}{2\mu_0} f_2\left(h,l,\lambda,\alpha,p\right)\frac{dp}{dx}\right]=\frac{\partial}{\partial x}\left(\frac{2\lambda\mu_0+e^{-\alpha p}h}{\lambda\mu_0+e^{-\alpha p}h}\cdot\frac{Uh}{2}\right)+\frac{\partial}{\partial x}\left(\frac{h^3 e^{-\alpha p}}{2\mu_0} f_2\left(h,l,\lambda,\alpha,p\right)F_{mx}\right) \quad (7)$$

Using the Finite Difference Method, we solved the Modified Reynolds Equation that includes variable viscosity, magnetic effect, slip-parameter, and the couple-stress fluid with the help the following non-dimensional parameters.

1.4 Non-Dimensional Variables and Parameters

$H=h/c$, $H_2=\frac{h_2}{c}$, $l^*=l/c$, $x=R\theta$, $A=\lambda\mu_0/c$, $p=\mu_0 URp^*/2c^2$, $\bar{\alpha}=\alpha p_s, h_m=h_{mo}H_m$,
$\alpha 1=\alpha^*=2\varsigma_0 X_m\left(h_{mo}\right)^2 c^2/R^2\mu_0 U$.

2. Bearing Characteristics

2.1 Load-Carrying Capacity

The load components (W) per unit length along the perpendicular to the line of centres are obtained by integrating the pressure around the bearing from $\theta=0$ to $\theta=\pi$ and the load components normal to the line of centres per unit length. The dimensionless load is given by:

$$W^*=\left[\begin{array}{l}\left\{\int_0^\pi\left[\frac{e^{\bar{\alpha}}}{H^3F_2\left(H,l^*,A,\bar{\alpha}\right)}\left(1+\frac{A}{A+He^{-\bar{\alpha}}}\right)H-\frac{e^{\bar{\alpha}}}{H^3F_2\left(H,l^*,A,\bar{\alpha}\right)}\left(1+\frac{A}{A+H_2e^{-\bar{\alpha}}}\right)H_2\right]\cos\theta d\theta\right\}^2+\\ \left\{\int_0^\pi\left[\frac{e^{\bar{\alpha}}}{H^3F_2\left(H,l^*,A,\bar{\alpha}\right)}\left(1+\frac{A}{A+He^{-\bar{\alpha}}}\right)H-\frac{e^{\bar{\alpha}}}{H^3F_2\left(H,l^*,A,\bar{\alpha}\right)}\left(1+\frac{A}{A+H_2e^{-\bar{\alpha}}}\right)H_2\right]\sin\theta d\theta\right\}^2\end{array}\right]^{1/2} \quad (8)$$

2.2 Attitude Angle (ψ) and Frictional Parameter (μ_f)

The angular position of the line of action for the load with respect to the location of the minimum film thickness, or point of closest approach, is the attitude angle ψ and the frictional parameter μ_f which may be determined from:

$$\tan\psi=\frac{W_0^*}{W_{\pi/2}^*}, \qquad \mu_f\left(R/C\right)=F^*/W^* \quad (9)$$

Where

$$F^* = \int_0^{\pi} \left(\frac{e^{\bar{\alpha}}}{H^3 F_2\left(H, l^*, A, \bar{\alpha}\right)} \left(1 + \frac{A}{A + He^{-\bar{\alpha}}}\right) H - \frac{e^{\bar{\alpha}}}{H^3 F_2\left(H, l^*, A, \bar{\alpha}\right)} \left(1 + \frac{A}{A + H_2 e^{-\bar{\alpha}}}\right) H_2 \right) \times \left(\frac{e^{-\bar{\alpha}} H \left(H - 2l^* e^{-\bar{\alpha}} \tanh\left(\frac{e^{-0..5\bar{\alpha}}}{2l^*} H \right) \right)}{2\left(A + e^{-\bar{\alpha}} H\right)} \right) d\theta - \int_{\pi}^{2\pi} \frac{1}{A + e^{-\bar{\alpha}} H} d\theta$$

3. Results and Discussion

The results shown in Fig. 1 to Fig. 8 and table 1, give the dimensionless pressure distribution (P) in the circumferential direction (θ), the variation for the dimensionless load carrying capacity (W), the attitude angle (ψ) and the modified friction coefficient (μ_f). We vary the eccentricity-ratio (ε) , the piezo-viscosity parameter (α), the slip-velocity (A), magnetic parameter $(\alpha 1)$, distance ratio (K) and the couple-stress parameter (l). When $l = 0$ the carrier liquid is considered as Newtonian but when $l \neq 0$ it is non-Newtonian. As $A \to 0$ the slip-velocity is maximum but when $A \to \infty$ we experience the no-slip condition. Eccentricity ratio is defined as $\varepsilon = \in / C$ where $\in$ and C is the eccentricity and clearance. The values of α is taken in the interval [0, 0.315] , the distance ratio which is the distance of the magnetic field over the radius of the bearing range from $K = 1.1 - 1.4$, while the values of the magnetic parameter range from $\alpha 1 = 0$ to $\alpha 1 = 8$.

3.1 Pressure Distribution

Table 1

Piezo-Viscosity Parameter α	Maximum Magnetic Pressure ($\theta = 90.63°$)	Maximum Hydrodynamic Pressure ($\theta = 151.9°$)
0.00	36.71	46.80
0.10	37.10	47.50
0.20	37.54	48.27
0.30	38.02	49.11

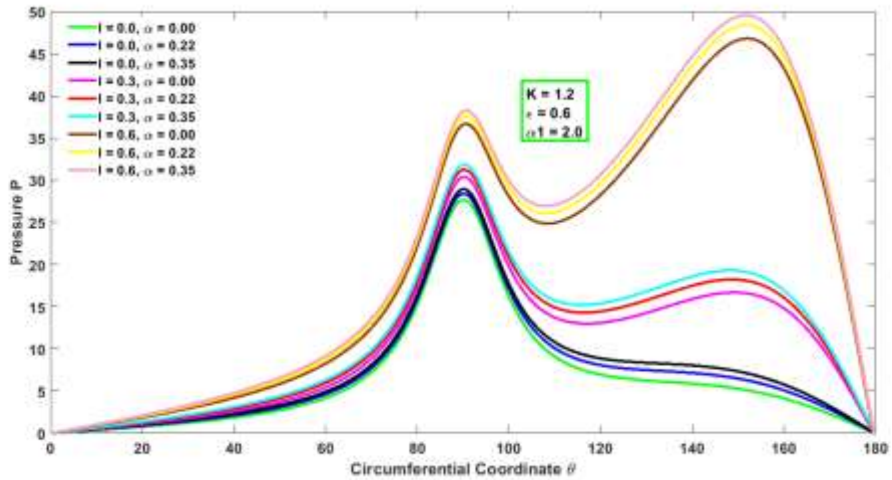

Figure 2

Figure 2 displays the pressure versus the circumferential coordinate θ, with slip-parameter $A=100$, piezo-viscosity parameter $\alpha=0.35$, distance ratio $K=1.2$ and magnetic parameter $\alpha 1=2$. The result shows two maximum pressure points. The first maximum occurred at $\theta=90.91°$ and the second at $\theta=150.9°$, which is due to the magnetic and hydrodynamic (HD) effects respectively. The second maximum depends on the couple-stress parameter (l), according to Nada et al. (2012). Additionally, for $l<0.48$, the first maximum pressure (magnetic) is greater than the HD pressure. Whereas, for $l>0.48$, the effect is reversed. The lowest pressure distribution occurs for $l=0$, but for an increase in l, we observe an increase in both the magnetic and HD pressure maximum. Table 1 shows the effect of the variation of the piezo-viscosity parameter on the maximum pressure. It is observed that α increases the maximum pressure.

Figure 3 represents the dimensionless load (W) versus Couple-stress parameter (l) with slip-velocity (A), magnetic $(\alpha 1=2)$, non-magnetic $(\alpha 1=0)$ lubricant and varying piezo-viscosity parameter (α). The load-carrying capacity of the journal bearing increases as l increases, where the least value of the load occurs when $l=0$ and the greatest value occurs, for $l=0.8$. We observe that the magnetic parameter increases the load-carrying capacity of the journal bearing.

An increase in the piezo-viscosity parameter increases the range of the load-carrying capacity. The greatest load-carrying ability occurs for high couple-stress parameter, piezo-viscosity parameter, together with the magnetic parameter incorporated into the lubricant. Figure 4 displays the result for load (W) vs. the slip-parameter (A) with varying couple-stress parameter (l) in the presence and absence of the magnetic parameter $(\alpha 1)$. As the slip-parameter increases, the load-carrying capacity of the journal bearing decreases; in the presence and absence of $\alpha 1$. The load capacity is more significant in the presence of the magnetic lubricant than the non-magnetic fluid.

3.2 Load-Carrying Capacity

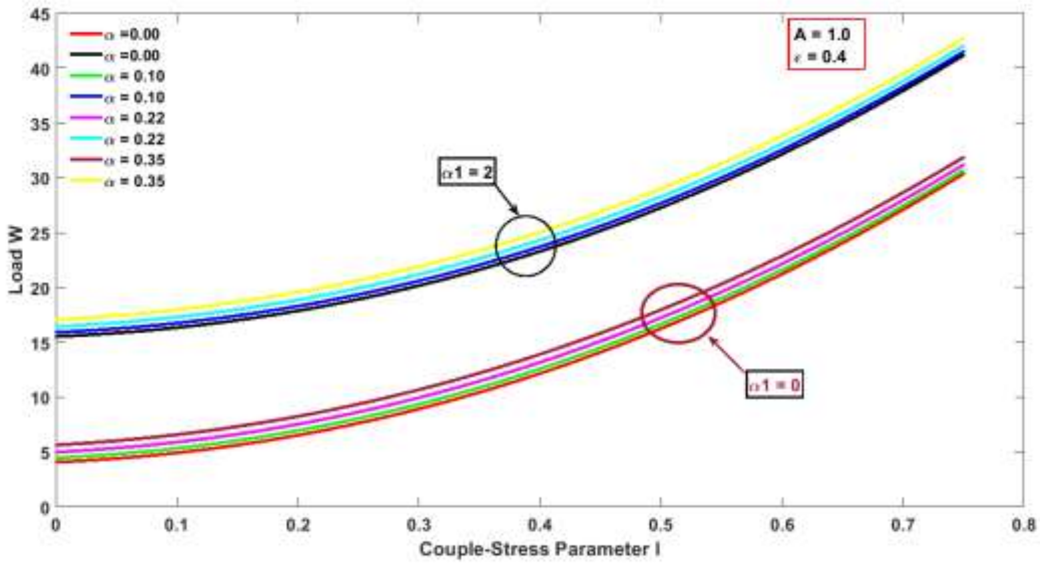

Figure 3

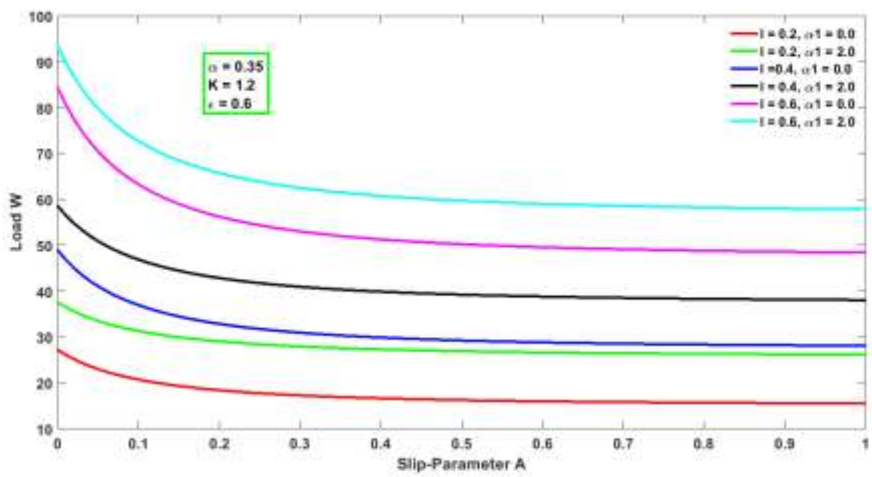

Figure 4

Since the magnetic lubricant gives the system an extra force, this additional force, together with the HD force, increases the load-carrying ability of the bearing. The greatest load is achieved when the couple-stress parameter is greatest, while in the presence of the magnetic parameter, with $A = 0$. Figure 5 focuses on the load (W) vs. the distance ratio (K), with increasing magnetic parameter $(\alpha 1)$ and eccentricity ratio (ε). As the distance ratio increases, the load-carrying capacity of the journal bearing decreases. The magnetic effect is reduced as the magnetic field is further away from the magnetic lubricant. This increased K cause a decrease in the magnetic field intensity, and the magnetic effect is therefore minimized. For any further increase the load remains constant. The greater the magnetic parameter, the greater is the load-carrying capacity of the journal bearing.

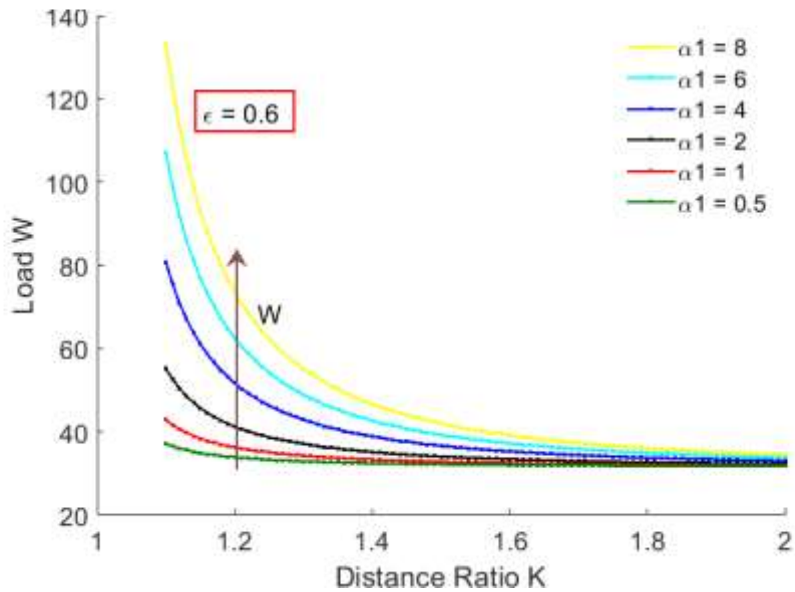

Figure 5

3.3 Friction Parameter

Figure 6 portrays the Frictional Parameter (μ_f) vs. the Distance Ratio (K) with varying Magnetic Parameter $(\alpha 1)$. For an increase in the distance ratio, friction increases, and the greatest friction occurs when $K = 1.4$. Also, for $K < 1.22$ the friction decreases with increasing $\alpha 1$, but for $K > 1.22$ the reverse happens. We also observed that at $K = 1.1$ for any increase in $\alpha 1$ there is a decrease in the friction, but at $K = 1.4$, when the magnetic parameter is increased the friction increases. This action is reversed when $K = 1.1$. Figure 7 illustrates the results of the frictional parameter (μ_f) vs. the couple-stress parameter (l) with varying magnetic parameter $(\alpha 1)$. As l increases the friction decreases with the lowest friction occurring when $l = 0.8$. As the magnetic parameter is increased the friction decreases, with the largest decrease occurring when $\alpha 1 = 8$, for $l < 0.43$, but when $l > 0.43$ the least friction occurs when $\alpha 1 = 2$ and with $\alpha 1 = 8$ the friction is greater. Hence, for least friction we would need high couple-stress and low magnetic parameter.

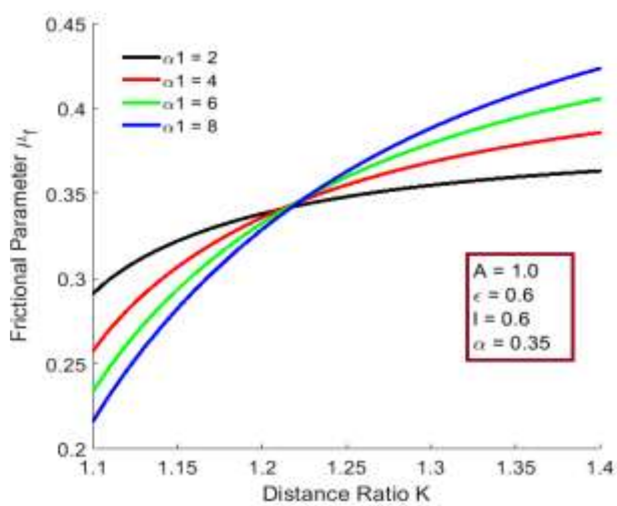

Figure 6

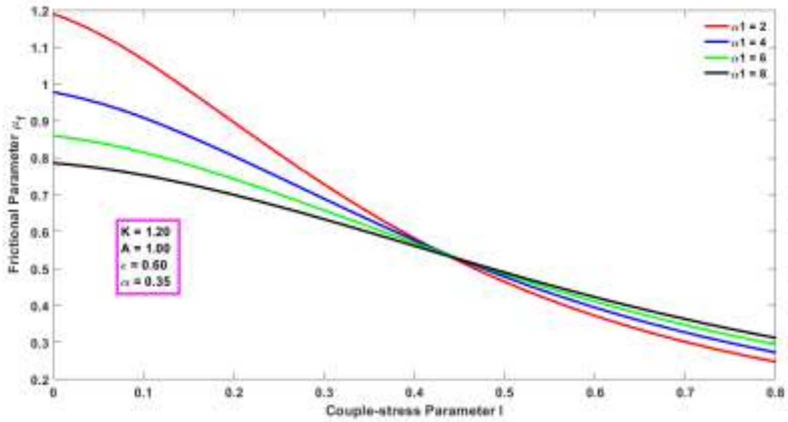

Figure 7

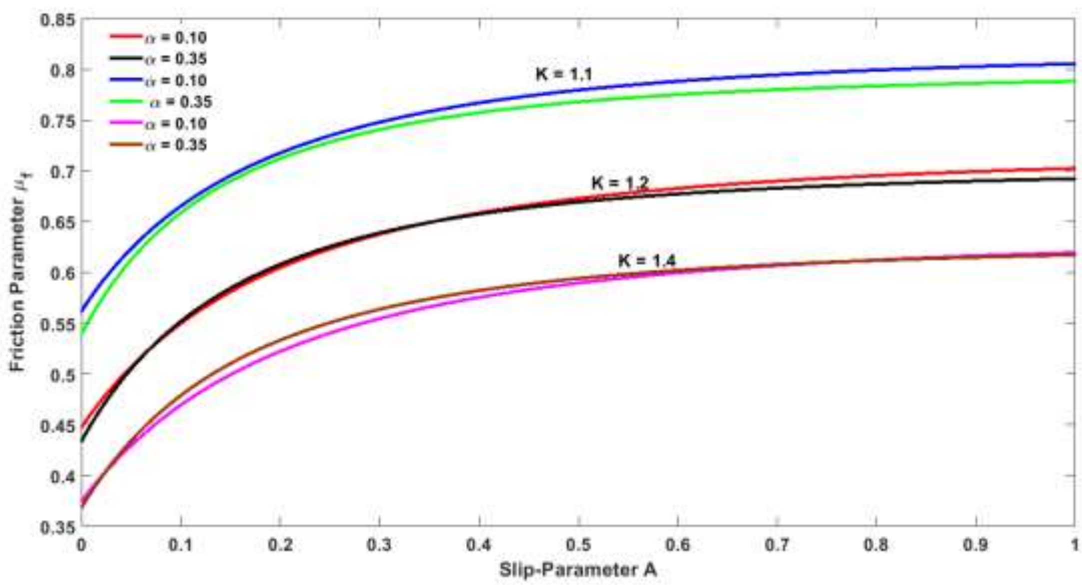

Figure 8

Figure 8 describes the Frictional Parameter (μ_f) versus the Slip-Parameter A with varying Distance ratio K and Piezo-viscosity Parameter α, a fixed value of $l = 0.4$ and $\alpha 1 = 2$ was used. As A becomes larger friction increases, this increase is modified with a reduced distance ratio (K). When $K = 1.1$, an increase in the piezo-viscosity parameter causes a small decrease in the frictional parameter. At $K = 1.4$ the friction is higher when $\alpha = 0.35$ from $0.024 < A < 0.82$ but less otherwise. Lastly, when $K = 1.2$ and $\alpha = 0.35$ the friction is greater when $0.07 < A < 0.37$ but less otherwise. The combined effect of piezo-viscosity, distance ratio and increasing slip affects the friction on the journal bearing by either lowering or increasing the frictional coefficient.

4. Conclusion

In an attempt to characterize the ideal journal bearing, a combined effect of the couple-stress lubricant, magnetic fluid, velocity-slip, and the piezo–viscosity effect have been utilized. Previous results indicated that the magnetic parameter improves the load-carrying capacity and the friction of the journal bearing. With the incorporation of the couple-stress lubricant, variable viscosity, and the slip-velocity, there is further enhancement of these characteristics. The maximum magnetic and HD pressures increase with increasing piezo-viscosity parameter in the presence of the slip parameter. Higher load values occur in the presence of the magnetic parameter, high eccentricity ratio, and higher piezo-viscosity parameter. For an increase in the magnetic parameter the friction decreases. The combined effect of high couple-stress parameter and low piezo-viscosity parameter yields the lowest friction. From the above results, we can enhance the journal bearing parameters by combining the slip, piezo-viscosity, and couple-stress parameters. Thereby, the production of an ideal journal bearing with minimum friction and high load carrying capacity can be achieved.

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